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Editorial

Defining city size

To the mind's eye, cities are dense agglomerations where, historically, people have come together to trade and engage in diverse social relationships. In abstract generalization, they are considered points in space where the friction of distance that restricts our ability to relate to one another is minimized. Ever since cities emerged from our nomadic past some 10 000 years ago they have been dominated by a relatively high density of living where people crowd together. Until about 200 years ago their size was limited by how far we could walk or at least ride a horse and cities rarely reached a population of more than 1 million. When they did, as in Rome, they fell apart at the limits of known technology. But the industrial revolution, which sparked the exploitation of mechanical energy, changed all that, just as the current seamless transition from energy to information is destined to do so again in the current period. It took fifty years for the internal combustion engine to make an impact on mass travel through the railways and nearly another 100 to miniaturize the technology to the point where individual travel using the automobile was possible. Cities took a long time to physically reflect these changes and only by the late 20th century had the dense concentrations that dominated the past begun to change through sprawl, globalization, and even greater polarization in their cores. This transition to an urban society, where we will all be living in cities of one kind or another, will be complete by the end of this century (Batty, 2011a), but the shadow of the historical city will be with us for a long time yet. The physical form of the city must thus be seen increasingly as a series of accidental or even purposive events frozen in time, the products of earlier technologies and social preferences.

If people live in crowded conditions, cheek by jowl as in the medieval city for example, the area that is inhabited increases less slowly than population size. If the area is $A \sim r^2$ and population increases as the volume $P \sim r^3$, then we can express this positive allometry of the population as $P \sim A^{3/2}$. Of course, this is an idealization, but it is likely that in compact cities which are dominated by very slow movement, the tendency to such a relationship holds. If people begin to spread out, as they are able to do with personal and individualized rapid transportation, then it is likely that this superlinearity or positive allometry might degrade to linearity or even sublinearity or negative allometry. The implication is that, as cities come to rely more and more on fast transport, people spread and sprawl. In this way the average density of the city at time $D(t)$, given by $D(t) = P(t)/A(t) \sim A(t)^{a(t)}$, falls, with the parameter $a(t)$ falling too. We can of course test this hypothesis by estimating the parameter $a(t)$ from data at each cross section in time.

The first known estimate of the allometric parameter was made by Stewart in his 1947 *Science* article that introduced social physics. In an almost throwaway line, he stated that the parameter $1 + a(1940)$ for all US cities with populations greater than 2500 in 1940 was $4/3$. His analysis would not satisfy our audience in this day and age, for there was no statistical scrutiny of the robustness of his conclusion, discussion of the error bounds, or interpretation of its significance. Yet Stewart's work heralded in a number of studies of the allometric parameter, which until about 1980 demonstrated positive allometry. For the last thirty years, allometric studies have been fewer, but the recent ones we have conducted with US and UK data indicate strong negative allometry and, in assessing all the studies available, there is an implication that the allometric parameter $1 + a(t)$ has been falling over this period. In table 1 we have compiled much of the evidence of the allometry of population size with respect to the area of cities since 1940. We also have included two studies of ancient settlement patterns and these reinforce the fact that, the further back one goes, the stronger the positive allometry.

Table 1. Evidence of allometry between area and population size.

Date, t	Number of cities, $N(t)$	Allometric coefficient, $1 + a(t)$	r^2 [a]	Reference
~10000BCE	339 hunter-gatherer places	1.429	0.240	Hamilton et al, 2007
~3000BCE	18 archaeological sites	1.188	0.774	Naroll, 1962
1940	412 US cities	1.333	NR	Stewart, 1947
1950	defined urbanized areas: US cities	1.167	0.706	Boyce, 1963
1950	155 US cities	1.163	0.865	Nordbeck, 1965
1950	53 US cities: Bartholomew data	1.351	0.900	Woldenberg, 1973
1950	155 US cities	1.163	0.850	Woldenberg, 1973
1951	157 UK cities	1.333	0.757	Stewart and Warntz, 1958
1953	Chinese cities	0.725	NR	Lo and Welch, 1977
1960	Swedish Cities	1.506	0.960	Nordbeck, 1971
1960	51 Ontario Canadian cities	1.149	0.980	Maher and Bourne, 1969
1960	defined urbanized areas: US cities	1.160	0.757	Boyce, 1963
1960	213 US cities	1.136	0.846	Nordbeck, 1965
1960	213 US cities	1.282	0.860	Woldenberg, 1973
1960	89 US cities: Manvel data	1.136	0.610	Woldenberg, 1973
1960	212 US cities	1.156	0.830	Lee, 1989
1961	205 UK cities	1.193	NR	Jones, 1975
1965	Swedish cities	1.538	0.790	Nordbeck, 1971
1965	518 Japanese cities	1.087	0.941	Nordbeck, 1965
1969	Satellite US cities	1.136	NR	Tobler, 1969
1970	248 US cities	1.149	0.846	Veregin and Tobler, 1997
1970	252 US cities	1.145	0.845	Lee, 1989
1973	40 historic classical buildings	1.299	0.980	Bon, 1973
1980	366 US cities	1.163	0.774	Veregin and Tobler, 1997
1980	373 US cities	1.171	0.876	Lee, 1989
1990	70 East Anglian UK cities	1.043	0.903	Batty and Longley, 1994
1990	801 Southeast UK cities	0.808	0.756	Batty and Longley, 1994
2001	386 European cities	1.014	0.760	Fuller and Gaston, 2009
2001	67 UK cities	0.946	NR	Fuller and Gaston, 2009
2001	100 highest population density metropolitan UK local authorities	0.765	0.637	Ferguson (this editorial)
2005	355 US Standard Metropolitan Statistical Area cities	0.676	0.309	Batty (this editorial)
2006	30 archetypal buildings	1.235	NR ~ 1	Steadman, 2006
2007	212 countries	0.769	0.710	Zhang and Yu, 2010
2009	~3.5m London buildings	1.296	0.835	Batty et al, 2008
2009	~122K London commercial buildings	1.199	0.838	Batty et al, 2008

[a] NR means not reported.

Yet conclusions from this evidence are ambiguous. First, the idea that city population scales with geometric volume or something between geometric volume and area is controversial. Most cities in developed countries occupy on average two stories, with vertical transportation restricted largely to walking, except for dense city cores where buildings greater than five stories with elevator transport are more frequent. In fact, a long line of work on fractal cities begun by Batty and Longley (1994) has established that the fractal dimension of cities tends to be between 1 and 2, with most cities being between about 1.7 and 1.9. Fractal dimension does not measure the same thing as the allometric coefficient, but it is another way of gauging how space is filled. Second, as transportation technologies have generated ever-faster movement, cities have spread out while densities have fallen, and this implies that the areas over which populations reside have increased more than proportionately. But the most important issue is how cities and their areas are defined. As cities have got bigger and have sprawled mainly in tree-like (dendritic) morphologies, the administrative areas over which population is counted have got bigger more than proportionately and the space between buildings and individual neighbourhoods has increased commensurately. Thus, this artifact of measurement would inevitably lead to lower allometric coefficients, perhaps enough to lower these from positive allometry 100 years ago, when cities were strongly centred around dense cores, to negative allometry. This finding is consistent with falling average densities as cities have got bigger.

To an extent, these speculations are more convincing for an earlier age, an age when the ubiquitous automobile technology first replaced fixed-line movement, such as rail, and before then the horse-drawn carriage and walking city. We are now entering an era when it appears as though there will be considerably more constraint put on automated physical transport, and cities may begin to reflect this in terms of increasing densities. Already globalization, the increasing embedding of dense information technologies into core cities, and the need for ever-more intense face-to-face contact have led to massive polarization in the biggest city cores, even while those same cities are sprawling at ever-lower densities in their peripheries. Defining a boundary around such constellations is ever-more difficult and, given the global reach of many economic functions in world cities, such boundaries are increasingly irrelevant. However, as we argued in an earlier editorial this year (Batty, 2011b), there is a strong interest in defining ever-more cogently economies of scale that are associated with the increasing size of cities based on the longstanding Marshallian premise that economies of scale are related to city size more than proportionately, thus demonstrating that the attributes of such economies scale superlinearly with city size, thus showing positive allometry.

Bettencourt et al (2007) have shown that several attributes of cities which reflect creative talents, ranging from patents to incomes, scale superlinearly with city size but our results concerning the more basic allometry of area versus population reflect the uncertainties of these particular conclusions. There are also doubts about the nature of the data and the estimation procedures for these kinds of allometric relationships (Shalizi, 2011; Warton et al, 2006), yet the crucial issue as before is what constitutes a city and what variables define size with respect to more creative pursuits. It is possible that the density of the core in terms of businesses, their variety as well as size, and the churn in terms of in-migration and out-migration might all be better measures to illustrate this superlinearity. However, the basic issue is the area over which the city is defined. As we all head to a world where everyone will live in some sort of city (Batty, 2011a), the definition of the appropriate unit is ever-more important. It is even possible that it is not location that is now the defining characteristic of city size but interaction, and only by studying this will be it be possible to track the global reach of any particular city. These pose enormous challenges for research. If we are to really grasp the conundrum that ever-larger cities are potentially more diverse, wealthier, greener, more liveable, and more innovative, we need to establish quite unambiguously the notion that bigness has this power. As cities get bigger, the opportunities for interaction get more than proportionately greater—as is seen from the simple idea that there are more potential interactions, assuming, of course, that urban populations can reach each other, and this depends on transport technology. So, in one sense, all of this follows. But the countervailing notion is the idea that, wherever one locates, one has global reach and that location in big cities is no longer as significant as it was. What we need is a sustained attack on this question of the geographical/geometric areas over which key variables defining scale and size should be measured, and this must now extend to interactions. In short, what is required is a new definition of the city, as a contact system, as a set of interactions and flows that define the kinds of network that enable creativity and innovation to thrive and grow. This is a challenge that now defines the way we must think about all cities.

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